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Abstract. In this paper we consider a controlled dynamical system consisting of two objects in the presence of external perturbations. The dynamics of the first object (the main object of the system) and the second object (the auxiliary object of the system) are described respectively by linear and nonlinear discrete-time recurrent vector equations. In the system under consideration there are two levels of control: basic level (the level I) that is dominant level and auxiliary level (the level II) that is subordinate level. The quality of the control process on every level of the control system is estimated by corresponding convex terminal functional. Both the control levels have different informational and control connections defined in advance. In this paper we study the problem of optimization of guaranteed result for program control the final state of phase vectors of the objects in the presence of external perturbations. For this problem we propose a mathematical formalization of two-level hierarchical minimax program control the final state of the dynamical system in the presence of external perturbations and incomplete information. In this paper for solving of the investigated problem is proposed the algorithm that has a form of a recurrent procedure of solving a linear and a convex mathematical programming problems, and a finite optimization problems.

INTRODUCTION

In this paper we consider a controlled dynamical system consisting of two objects in the presence of external perturbations. The dynamics of the first object (the main object of the system) and the second object (the auxiliary object of the system) are described respectively by linear and nonlinear discrete-time recurrent vector equations. In the system under consideration there are two levels of control: basic level (the level I) that is dominant level and auxiliary level (the level II) that is subordinate level. The quality of the control process on every level of the control system is estimated by corresponding convex terminal functional. Both the control levels have different informational and control connections defined in advance. In this paper we study the problem of optimization of guaranteed result for program control the final state of phase vectors of the objects in the presence of external perturbations. For this problem we propose a mathematical formalization of two-level hierarchical minimax program control the final state of the dynamical system in the presence of external perturbations and incomplete information. In this paper for solving of the investigated problem is proposed the algorithm that has a form of a recurrent procedure of solving a linear and a convex mathematical programming problems, and a finite optimization problems.

Results obtained in this article are based on the studies [1]–[5] and can be used for computer simulation, design and construction of multilevel control systems for actual economic, technical and other dynamical processes operating under deficit of information and uncertainty. Mathematical models of such systems are presented, for example, in works [1]–[3], [6]–[9].

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OBJECT'S DYNAMICS IN THE TWO-LEVEL HIERARCHICAL CONTROL SYSTEM

On a given integer-valued time interval (simply interval) $\overline{0,T} = \{0, 1, \cdots, T\}$ ($T > 0,\ T \in \mathbb{N}$; where $\mathbb{N}$ is the set of all natural numbers) we consider a controlled multistep dynamical system which consists of the two objects. Dynamics of the object I (main object of the system) controlled by dominant player P, is described by the vector linear discrete-time recurrent relation of the form

$$y(t+1) = A(t)y(t) + B(t)u(t) + C(t)v(t) + D(t)\xi(t)),\ y(0) = y_0, \tag{1}$$

and the dynamics of the object II (auxiliary object of the system) controlled by subordinate player E, is described by the vector nonlinear discrete-time recurrent relation:

$$z(t+1) = f(t, z(t), u(t), v(t), \xi^{(1)}(t)),\ z(0) = z_0, \tag{2}$$

where $t \in \overline{0, T-1}$; $y(t) = (y_1(t), y_2(t), \ldots, y_r(t)) \in \mathbb{R}^r$ is a phase vector of the object I at the time moment t; $z(t) = (z_1(t), z_2(t), \ldots, z_s(t)) \in \mathbb{R}^s$ is a phase vector of the object II at the time moment t; ($r, s \in \mathbb{N}$; for $n \in \mathbb{N}$, $\mathbb{R}^n$ is an n-dimensional Euclidean vector space of column vectors); $u(t) = (u_1(t), u_2(t), \ldots, u_p(t)) \in \mathbb{R}^p$ is a vector of control action (control) of the dominant player P at the time moment t, that satisfies the given constraint:

$$u(t) \in \mathbf{U}_1(t) \subset \mathbb{R}^p,\ \mathbf{U}_1(t) = \{u(t):\ u(t) \in \{u^{(1)}(t), u^{(2)}(t), \cdots, u^{(N_t)}(t)\} \subset \mathbb{R}^p\}, \tag{3}$$

where $\mathbf{U}_1(t)$ for each time moment $t \in \overline{0, T-1}$ is a finite set of vectors in the space $\mathbb{R}^p$, consisting of N_t ($N_t \in \mathbb{N}$) vectors in the space $\mathbb{R}^p$ ($p \in \mathbb{N}$); $v(t) = (v_1(t), v_2(t), \ldots, v_q(t)) \in \mathbb{R}^q$ is a vector of control action (control) of the subordinate player E at the time moment t, which depends on admissible realization of the control $u(t) = u^{(j)} \in \mathbf{U}_1(t)$ ($j \in \overline{1, N_t}$) of the player P and must be satisfy the given constraint:

$$v(t) \in \mathbf{V}_1(u(t)) \subset \mathbb{R}^q,\ \mathbf{V}_1(u(t)) = \{v(t):\ v(t) \in \{v^{(1)}(t), v^{(2)}(t), \cdots, v^{(Q_t(j))}(t)\} \subset \mathbb{R}^q\}, \tag{4}$$

where $\mathbf{V}_1(u(t))$ for each time moment $t \in \overline{0, T-1}$ and control $u(t) = u^{(j)} \in \mathbf{U}_1(t)$ of the player P is the finite set of vectors in the space $\mathbb{R}^q$, consisting of $Q_t(j)$ ($Q_t(j) \in \mathbb{N}$) vectors in the space $\mathbb{R}^q$ ($q \in \mathbb{N}$).

In the equations (1) and (2) describing dynamics of the objects I and II, respectively, $\xi(t) = (\xi_1(t), \xi_2(t), \ldots, \xi_m(t)) \in \mathbb{R}^m$ and $\xi^{(1)}(t) = (\xi_1^{(1)}(t), \xi_2^{(1)}(t), \ldots, \xi_l^{(1)}(t)) \in \mathbb{R}^l$ are a perturbations vectors for these objects that at each time moment t ($t \in \overline{0, T-1}$) satisfies the given constraints:

$$\xi(t) \in \Xi_1(t) \subset \mathbb{R}^m,\ \xi^{(1)}(t) \in \Xi_1^{(1)}(t) \subset \mathbb{R}^l, \tag{5}$$

where the sets $\Xi_1(t)$ and $\Xi_1^{(1)}(t)$ are convex, closed and bounded polyhedrons (with a finite number of vertices) in the spaces $\mathbb{R}^m$ and $\mathbb{R}^l$, respectively and restrict admissible values of realizations of perturbations vectors of the objects I and II, respectively at the time moment t.

We assume, that for all fixed $t \in \overline{0, T-1}$ all matrixes $A(t)$, $B(t)$, $C(t)$, and $D(t)$ in a vector recurrent equation (1), describing dynamics of the object I, are real matrices of dimensions $(r \times r)$, $(r \times p)$, $(r \times q)$, and $(r \times m)$, respectively; the vector-function $f : \overline{0, T-1} \times \mathbb{R}^s \times \mathbb{R}^p \times \mathbb{R}^q \times \mathbb{R}^l \longrightarrow \mathbb{R}^s$ in a vector recurrent equation (2), describing dynamics of the object II is continuous by collection of the variables $(z(t), u(t), v(t), \xi^{(1)}(t))$; for all fixed time moment $t \in \overline{0, T-1}$, and convex set $Z_* \subset \mathbb{R}^s$, and controls $u_*(t) \in \mathbf{U}_1(t)$ and $v_*(t) \in \mathbf{V}_1(u_*(t))$, the set $f(t, Z_*, u_*(t), v_*(t), \Xi_1^{(1)}) = \{f(t, z(t), u_*(t), v_*(t), \xi^{(1)}(t)),\ z(t) \in Z_*,\ \xi^{(1)}(t) \in \Xi_1^{(1)}\}$ is convex set of the space $\mathbb{R}^s$.

INFORMATION CONDITIONS FOR THE PLAYERS IN THE CONTROL SYSTEM

The control process in discrete-time dynamical system (1)–(5) are realized in the presence of the following information conditions.

It is assumed that in the field of interests of the player P are both possible terminal (final) states $y(T)$ of the object I and possible states $z(T)$ of the object II, and for any considered time interval $\overline{\tau, T} \subseteq \overline{0, T}$ ($\tau < T$) the player P also knows a future realization of the program control $v(\cdot) = \{v(t)\}_{t \in \overline{\tau, T-1}}$ ($\forall\ t \in \overline{\tau, T-1}:\ v(t) \in \mathbf{V}_1(u^{(j)}(t))$,

$u^{(j)}(t) \in \mathbf{U}_1(t), j \in N_t$) of the player E at this time interval which communicate to him, and he can use its for constructing his program control $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,T-1}}$ $(\forall\ t \in \overline{\tau, T-1} :\ u(t) \in \mathbf{U}_1(t))$.

We assumed that in the field of interests of the player E are only possible terminal states $z(T)$ of the object II and for any considered time interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$ he also knows a future realization of the control $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,T-1}}$ $(\forall\ t \in \overline{\tau, T-1} :\ u(t) \in \mathbf{U}_1(t))$ of the player P at this time interval, which communicate to him, and he can use its for constructing his program control $v(\cdot) = \{v(t)\}_{t\in\overline{\tau,T-1}}$ $(\forall\ t \in \overline{\tau, T-1} :\ v(t) \in \mathbf{V}_1(u^{(j)}(t)),\ u^{(j)}(t) \in \mathbf{U}_1(t), j \in N_t)$. Therefore, the behavior of player E explicitly depends on the behavior of player P.

It is also assumed that in the considered control process for every instant $t \in \overline{0, T}$ players P and E knows all relations and constraints (1)–(5).

Then on the basis of given assumptions we will say that such possibilities of the behavior of player P combined with the player E, and objects I and II are defined as the level I or the dominant level of the control process in considered system (1)–(5).

The player E and object II controlled by them form the level II or the subordinate level of control in considered system (1)–(5) (which is subordinate to the level I or the dominating level of the control process).

It is assumed that the player P estimate the result of the realization of this control process (1)–(5) by the values of the linear functional $\hat{\alpha} :\ \mathbb{R}^r \times \mathbb{R}^s \to \mathbb{R}^1$, which is defined on the final (terminal) phase states $y(T)$ and $z(T)$ of the objects I and II, respectively.

The aim of player P on the level I of this control process and fixed time interval $\overline{t, T} \subseteq \overline{0, T}$ $(t < T)$ can be formulate in the following way. The player P using his information and control possibilities has interest in such result of control process in dynamical system (1)–(5) on the interval $\overline{t, \mathrm{T}}$ when functional $\hat{\alpha}$ has minimal admissible value at worst for him realization of perturbation vectors $\xi(\cdot) = \{\xi(t)\}_{t\in\overline{\tau,T-1}}$ and $\xi^{(1)}(\cdot) = \{\xi^{(1)}(t)\}_{t\in\overline{\tau,T-1}}$. And this aim he can realize by the way a choice his program control $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,T-1}}$ $(\forall\ t \in \overline{\tau, T-1} :\ u(t) \in \mathbf{U}_1(t))$ and on the base of program control $v(\cdot) = \{v(t)\}_{t\in\overline{\tau,T-1}}$ $(\forall\ t \in \overline{\tau, T-1} :\ v(t) \in \mathbf{V}_1(u(t)),\ u(t) \in \mathbf{U}_1(t))$ of the player E at this interval, which communicate to him. Note, that the player E helps to him in achieving its aim.

It is assumed that the player E estimate the result of the realization of this control process (1)–(5) by the values of the convex functional $\hat{\beta} :\ \mathbb{R}^s \to \mathbb{R}^1$, which is defined on the final (terminal) phase states of the object II.

Then the aim of the player E on the level II of this control process and fixed interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$ can be formulate in the following way. The player E using his information and control possibilities has interest in such result of control process in dynamical system (1)–(5) on the interval $\overline{\tau, \mathrm{T}}$ when linear functional $\hat{\beta}$ has minimal admissible value at worst for him realization of perturbation vector $\xi^{(1)}(\cdot) = \{\xi^{(1)}(t)\}_{t\in\overline{\tau,T-1}}$. And this aim he can realize by the way a choice his program control $v(\cdot) = \{v(t)\}_{t\in\overline{\tau,T-1}}$ $(\forall\ t \in \overline{\tau, T-1} :\ v(t) \in \mathbf{V}_1(u(t)),\ u(t) \in \mathbf{U}_1(t))$ on the base of program control $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,T-1}}$ $(\forall\ t \in \overline{\tau, T-1} :\ u(t) \in \mathbf{U}_1(t))$ of the player P at this time interval, which communicate to him.

DEFINITIONS AND CRITERIONS OF QUALITY FOR THE CONTROL PROCESS

For a strict mathematical formulation the two-level hierarchical minimax program control problem by a final states phase vectors in discrete-time dynamical system (1)–(5) with perturbation we introduce some definitions.

For a fixed number $k \in \mathbb{N}$ and an integer-valued interval $\overline{\tau, \vartheta} \subseteq \overline{0, T}$ $(\tau \leq \vartheta)$, we denote by $\mathbf{S}_k(\overline{\tau, \vartheta})$ the metric space of functions $\varphi :\ \overline{\tau, \vartheta} \longrightarrow \mathbb{R}^k$ of an integer argument t where the metric ρ_k is defined as

$$\rho_k(\varphi_1(\cdot), \varphi_2(\cdot)) = \max_{t\in\overline{\tau,\vartheta}} \parallel \varphi_1(t) - \varphi_2(t) \parallel_k \ \ ((\varphi_1(\cdot), \varphi_2(\cdot)) \in \mathbf{S}_k(\overline{\tau, \vartheta}) \times \mathbf{S}_k(\overline{\tau, \vartheta}));$$

by $\mathrm{comp}(\mathbf{S}_k(\overline{\tau, \vartheta}))$ we denote the set of all nonempty and compact (in the sense of this metric) subsets of the space $\mathbf{S}_k(\overline{\tau, \vartheta})$. Here for $x \in \mathbb{R}^k$ in what follows $\parallel x \parallel_k$ denotes the Euclidean norm of vector x in the space $\mathbb{R}^k$.

Based on constraint (3) we define the set $\mathbf{U}(\overline{\tau, \vartheta}) \in \mathrm{comp}(\mathbf{S}_p(\overline{\tau, \vartheta - 1}))$ of all admissible program controls $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,\vartheta-1}}$ of the player P on the interval $\overline{\tau, \vartheta} \subseteq \overline{0, T}$ $(\tau < \vartheta)$ with relation

$$\mathbf{U}(\overline{\tau, \vartheta}) = \{u(\cdot) :\ u(\cdot) \in \mathbf{S}_p(\overline{\tau, \vartheta - 1}),\ \forall\ t \in \overline{\tau, \vartheta - 1},\ u(t) \in \mathbf{U}_1(t)\}.$$

Similarly, for a fixed program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \vartheta})$ of the player P according to constraint (4) we define the set $\mathbf{V}(\overline{\tau, \vartheta}; u(\cdot))$ of all admissible program controls of player E on the interval $\overline{\tau, \vartheta} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \vartheta)$ of the corresponding $u(\cdot)$, by the following relation

$$\mathbf{V}(\overline{\tau, \vartheta}; u(\cdot)) = \{v(\cdot) : \ v(\cdot) \in \mathbf{S}_q(\overline{\tau, \vartheta - 1}), \ \forall\, t \in \overline{\tau, \vartheta - 1}, \ v(t) \in \mathbf{V}_1(u(t))\}.$$

It should be noted that by virtue of (3) and (4) the $\mathbf{U}(\overline{\tau, \vartheta})$ and $\mathbf{V}(\overline{\tau, \vartheta}; u(\cdot))$ are finite sets in the corresponding vector spaces.

Analogy, according to constraints (5) we define the sets $\Xi(\overline{\tau, \vartheta})$ and $\Xi^{(1)}(\overline{\tau, \vartheta}; u(\cdot))$ of all admissible program perturbations vectors that respectively affect on the dynamics of the objects I and II on the interval $\overline{\tau, \vartheta} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \vartheta)$ by the following relations:

$$\Xi(\overline{\tau, \vartheta}) = \{\xi(\cdot) : \ \xi(\cdot) \in \mathbf{S}_m(\overline{\tau, \vartheta - 1}), \ \forall\, t \in \overline{\tau, \vartheta - 1}, \ \xi(t) \in \Xi_*\};$$

$$\Xi^{(1)}(\overline{\tau, \vartheta}) = \{\xi^{(1)}(\cdot) : \ \xi^{(1)}(\cdot) \in \mathbf{S}_l(\overline{\tau, \vartheta - 1}), \ \forall\, t \in \overline{\tau, \vartheta - 1}, \ \xi^{(1)}(t) \in \Xi_*^{(1)}\}.$$

Let for instant $\tau \in \overline{0, T}$ the set $\mathbf{W}(\tau) = \overline{0, T} \times \mathbf{R}^r \times \mathbb{R}^s$ is the set of all admissible τ-positions $w(\tau) = \{0, y(\tau), z(\tau)\} \in \overline{0, T} \times \mathbb{R}^r \times \mathbb{R}^s$ of the player P ($\mathbf{W}(0) = \{w(0)\} = \mathbf{W}_0 = \{w_0\}, \ w(0) = w_0 = \{0, y_0, z_0\}$) on level I of the control process.

Then, for any interval $\overline{\tau, T} \subset \overline{0, T}$, and admissible realizations of τ-position $w(\tau) \in \mathbf{W}(\tau)$, program controls $u(\cdot) \in \mathbf{U}(\overline{\tau, T})$ and $v(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot))$, and program perturbation vectors $\xi(\cdot) \in \Xi(\overline{\tau, T})$ and $\xi^{(1)}(\cdot) \in \Xi^{(1)}(\overline{\tau, T})$, for estimating from the point of view of the player P the quality of the control process on the level I we define the following convex terminal functional

$$\alpha : \ \mathbf{W}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times \hat{\mathbf{V}}(\overline{\tau, T}) \times \Xi(\overline{\tau, T}) \times \Xi^{(1)}(\overline{\tau, T}) = \mathbf{\Gamma}(\overline{\tau, T}, \alpha) \longrightarrow \mathbf{E} =]-\infty, +\infty[, \tag{6}$$

and its value for each collection $(w(\tau), u(\cdot), v(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) \in \mathbf{W}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times \hat{\mathbf{V}}(\overline{\tau, T}) \times \Xi(\overline{\tau, T}) \times \Xi^{(1)}(\overline{\tau, T})$ is defined by the following relation

$$\alpha(w(\tau), u(\cdot), v(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) = \hat{\alpha}(y(T), z(T)) = \mu \cdot < e, y(T) >_r + \mu^{(1)} \cdot \hat{\beta}(z(T)). \tag{7}$$

Where $\hat{\mathbf{V}}(\overline{\tau, T}) = \{\mathbf{V}(\overline{\tau, T}; u(\cdot)), \ u(\cdot) \in \mathbf{U}(\overline{\tau, T})\}$; by $y(T) = y_T(\overline{\tau, T}, y(\tau), u(\cdot), v(\cdot), \xi(\cdot))$, and by $z(T) = z_T(\overline{\tau, T}, z(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot))$ we denote the sections of motions of object I and object II, respectively at final (terminal) instant T on the interval $\overline{\tau, T}$; $\hat{\alpha} : \ \mathbb{R}^r \times \mathbb{R}^s \to \mathbb{R}^1$ is linear terminal functional; $\hat{\beta} : \ \mathbb{R}^s \to \mathbb{R}^1$ is convex terminal functional; $e \in \mathbb{R}^r$ is fixed vector; here and below, for each $k \in \mathbb{N}, \ a \in \mathbb{R}^k$ and $b \in \mathbb{R}^k$ will be denoted by the symbol $< a, b >_k$ scalar product of vectors a and b of the space $\mathbb{R}^k$; $\mu \in \mathbb{R}^1$ and $\mu^{(1)} \in \mathbb{R}^1$ are fixed numerical parameters which satisfying the following conditions:

$$\mu \geq 0; \ \mu^{(1)} \geq 0; \ \mu + \mu^{(1)} = 1. \tag{8}$$

We denote by $\mathbf{W}^{(1)}(\tau) = \overline{0, T} \times \mathbb{R}^s$ the set of all admissible τ-positions $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \overline{0, T} \times \mathbb{R}^s$ of the player E ($\mathbf{W}^{(1)}(0) = \{w^{(1)}(0)\} = \mathbf{W}_0^{(1)} = \{w_0^{(1)}\}, \ w^{(1)}(0) = w_0^{(1)} = \{0, z_0\}$) on level II of the control process.

Then, for any interval $\overline{\tau, T} \subset \overline{0, T}$, and admissible realizations of τ-position $w^{(1)}(\tau) \in \mathbf{W}^{(1)}(\tau)$, program controls $u(\cdot) \in \mathbf{U}(\overline{\tau, T})$ and $v(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot))$, and program perturbation vector $\xi^{(1)}(\cdot) \in \Xi^{(1)}(\overline{\tau, T})$, for estimating from the point of view of the player E the quality of the control process on the level II we define the following convex terminal functional

$$\beta : \ \mathbf{W}^{(1)}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times \hat{\mathbf{V}}(\overline{\tau, T}) \times \Xi^{(1)}(\overline{\tau, T}) = \mathbf{\Gamma}(\overline{\tau, T}, \beta) \longrightarrow \mathbf{E}, \tag{9}$$

which estimate for player E a quality of the final phase states of the object II, and its value for each collection $(w^{(1)}(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot)) \in \mathbf{W}^{(1)}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times \hat{\mathbf{V}}(\overline{\tau, T}) \times \Xi^{(1)}(\overline{\tau, T})$ is defined by the following relation

$$\beta(w^{(1)}(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot)) = \hat{\beta}(z(T)). \tag{10}$$

Where $\hat{\beta} : \ \mathbb{R}^s \to \mathbb{R}^1$ is convex terminal functional; $z(T) = z_T(\overline{\tau, T}, z(\tau), u(\cdot), \ v(\cdot), \xi^{(1)}(\cdot))$ is the section of motion of object II at final (terminal) instant T on the interval $\overline{\tau, T}$; $e^{(1)} \in \mathbb{R}^s$ is fixed vector.

Let also, for any interval $\overline{\tau, T} \subset \overline{0, T}$, and admissible realizations of τ-position $w(\tau) \in \mathbf{W}(\tau)$, program controls $u(\cdot) \in \mathbf{U}(\overline{\tau, T})$ and $v(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot))$, and program perturbation vector $\xi(\cdot) \in \Xi(\overline{\tau, T})$ we shall consider the linear terminal functional

$$\gamma : \ \mathbf{W}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times \hat{\mathbf{V}}(\overline{\tau, T}) \times \Xi(\overline{\tau, T}) = \mathbf{\Gamma}(\overline{\tau, T}, \gamma) \longrightarrow \mathbf{E}, \tag{11}$$

which estimate for player P a quality of the final phase states of the object I, and its value for each collection $(w(\tau), u(\cdot), v(\cdot), \xi(\cdot)) \in \mathbf{W}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times \hat{\mathbf{V}}(\overline{\tau, T}) \times \mathbf{\Xi}(\overline{\tau, T})$ is defined by the following relation

$$\gamma(w(\tau), u(\cdot), v(\cdot), \xi(\cdot)) = \hat{\gamma}(y(T)) =< e, y(T) >_r . \tag{12}$$

Where $\hat{\gamma}$ is linear terminal functional from (7); $y(T) = y_T(\overline{\tau, T}, y(\tau), u(\cdot), v(\cdot), \xi(\cdot))$ is the section of motion of object I at final (terminal) instant T on the interval $\overline{\tau, T}$.

Then if we also consider the vector-functional $\delta = (\gamma, \beta)$ such that it define by relation

$$\delta : \ \mathbf{\Gamma}(\overline{\tau, T}, \gamma) \times \mathbf{\Gamma}(\overline{\tau, T}, \beta) \longrightarrow \mathbf{E}^2, \tag{13}$$

and its two values for admissible on the interval $\overline{\tau, T}$ realizations of all arguments are defined according to relations (9)–(12) and we can assert that functional α, which is defined by relations (6)–(8) is its convolution after using the scalar's method for vector functionals [5].

OPTIMIZATION PROBLEMS FOR THE CONTROL PROCESS

On the basis of the assumptions made, the aim of the player E in the program control process on fixed interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$ can be formulated in the following way. The player E using his information and control possibilities has interest in such result of control process in dynamical system (1)–(5) on the interval $\overline{\tau, T}$ when functional β which determined by relations (9) and (10) for each admissible realizations of his τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ and program control $u(\cdot) \in \mathbf{U}(\overline{\tau, T})$ of the player P on it interval has minimal admissible value by means way using to choice his admissible program control $v(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot))$. We note, that the following situation is not excluded from the analysis, when the program perturbation $\xi^{(1)}(\cdot) \in \mathbf{\Xi}^{(1)}(\overline{\tau, T})$ can be realized in the worst way for the player E, i.e. defining the most possible value of the functional β under the fixed realizations of the parameters $w^{(1)}(\tau)$, $u(\cdot)$ and $v(\cdot)$.

Then for realization it aim of the player E we can formulate the following minimax program control problem by a final state phase vector of the object II on the level II of the control in two-level hierarchical control process for dynamical system (1)–(5).

Problem 1. For fixed interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$, admissible on the level II in the two level hierarchical control system for dynamical system (1)–(5) realization τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ of the player E and every admissible realization of the program control $u(\cdot) \in \mathbf{U}(\overline{\tau, T})$ of the player P on the level I of this control process, it is required to find the set $\mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) \subseteq \mathbf{V}(\overline{\tau, T}; (u(\cdot))$ of minimax program controls $v^{(e)}(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot))$ of the player E corresponding the control $u(\cdot)$ of the player P and it set is determine by the following relation:

$$\mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) = \{v^{(e)}(\cdot) : \ v^{(e)}(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot)), \ c_\beta^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) =$$

$$= \max_{\xi^{(1)}(\cdot) \in \ \mathbf{\Xi}^{(1)}(\overline{\tau, T})} \beta(w^{(1)}(\tau), v^{(e)}(\cdot), u(\cdot), \xi^{(1)}(\cdot)) = \min_{v(\cdot) \in \ \mathbf{V}(\overline{\tau, T}; u(\cdot))} \ \max_{\xi^{(1)}(\cdot) \in \ \mathbf{\Xi}^{(1)}(\overline{\tau, T})} \beta(w^{(1)}(\tau), v(\cdot), u(\cdot), \xi^{(1)}(\cdot))\}, \tag{14}$$

where the functional β is defined by the relations (9) and (10).

We call the set $\mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ which formed due from solving of the problem 1 — the set of minimax program controls of the player E on the level II of the control of this two level hierarchical control process for the dynamical system (1)–(5) and corresponding to it the numeric value $c_\beta^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ we call as the value of the result of the minimax program control of the player E on the level II of the control for this control process.

At the corresponding of the definitions and assumptions made above about parameters and information relations for the dynamical system (1)–(5), the aim of the player P which combined with the player E and objects I and II define the level I of this control process on the interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$, can be formulated in the following way. The player P using their information and controls possibilities interested in such result of realization the program two-level control process for this dynamical system on the interval $\overline{\tau, T}$ when functional α determined by relations (6) – (8) for each admissible his τ-position $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = \{0, y_0, z_0\} = w_0 \in \mathbf{W}_0)$ has minimal admissible value using choice their admissible program control $u(\cdot) \in U(\overline{\tau, T})$ and program minimax control $v^{(e)}(\cdot) \in \mathbf{V}^{(e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot))$ of the player E from solving the problem 1 (where τ-position $w^{(1)}(\tau) \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) =$

$\{0, z_0\} = w_0 \in \mathbf{W}_0^{(1)}$) and determines the phase state of object II on the level II of the control process at instant τ and it is form from τ-position $w(\tau)$). We note, that the following situation is not excluded from the analysis, when the program perturbations $\xi(\cdot) \in \Xi(\overline{\tau, T})$ and $\xi^{(1)}(\cdot) \in \Xi^{(1)}(\overline{\tau, T})$ can be realized in the worst way for the player P, i.e. defining the most possible value of the functional α under the fixed realizations of the parameters $w(\tau)$, $w^{(1)}(\tau)$, $u(\cdot)$ and $v(\cdot)$.

Below, for realization it aim of the player P corresponding by the level I of considered control process we formulate the following minimax program control problem by a final state phase vectors of the objects I and II on the level I of the control in two-level hierarchical control process for dynamical system (1)–(5).

Problem 2. For fixed time interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$ and admissible on the level I of the two-level hierarchical dynamical system (1)–(5) of the realization τ-position $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = \{0, y_0, z_0\} = w_0 \in \mathbf{W}_0)$ of the player P it is required to find the set $\mathbf{U}^{(e)}(\overline{\tau, T}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau, T})$ of the minimax program controls of the player P which determine by the following relation

$$\mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) = \{u^{(e)}(\cdot) : \ u^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau, T}),$$

$$c_\alpha^{(e)}(\overline{\tau, T}, w(\tau)) = \min_{v^{(e)}(\cdot) \in \mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot))} \ \max_{\substack{\xi(\cdot) \in \Xi(\overline{\tau, T}; u^{(e)}(\cdot)) \\ \xi^{(1)}(\cdot) \in \Xi^{(1)}(\overline{\tau, T})}} \alpha(w(\tau), u^{(e)}(\cdot), v^{(e)}(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) =$$

$$= \min_{u(\cdot) \in \mathbf{U}(\overline{\tau, T})} \ \min_{v^{(e)}(\cdot) \in \mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))} \ \max_{\substack{\xi(\cdot) \in \Xi(\overline{\tau, T}) \\ \xi^{(1)}(\cdot) \in \Xi^{(1)}(\overline{\tau, T})}} \alpha(w(\tau), u(\cdot), v^{(e)}(\cdot), \xi(\cdot), \xi^{(1)}(\cdot))\}. \tag{15}$$

Where the functional α defined by relations (6) – (8); τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = \{0, z_0\} = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ of the player E formed due from τ-position $w(\tau)$ of the player P and determines the realization at instant τ the phase state object II on the level II of this control process for dynamical system (1)–(5) and the set $\mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) = \{v^{(e)}(\cdot)\} \subseteq \mathbf{V}(\overline{\tau, T})$ minimax program controls of the player E for level II of the control in this control process for any realizations τ-position $w^{(1)}(\tau) \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ of the player E and program control $u(\cdot) \in \mathbf{U}(\overline{\tau, T})$ of the player P which formed due from solving of the problem 1.

Then the set $\mathbf{U}^{(e)}(\overline{\tau, T}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau, T})$ which is forming from solving of the problems 1 and 2 we call the set of minimax program controls of the player P on the level I of the control of the two-level hierarchial control process for dynamical system (1)–(5) and corresponding to it the number $c_\alpha^{(e)}(\overline{\tau, T}, w(\tau))$ we call the value of the result of the minimax program control for player P on the level I of this control process.

Based on the solution of the problems 1 and 2 we consider the following problem.

Problem 3. For fixed time interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$ and admissible on the level I of the control in two-level hierarchical dynamical system (1)–(5) realization the τ-position $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = \{0, y_0, z_0\} = w_0 \in \mathbf{W}_0)$ of the player P and admissible on the level II of the control process for this dynamical system the realization τ-position $w^{(1)}(\tau) \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ of the player E which formed due from the τ-position $w(\tau)$, and admissible realization of the program minimax control $u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau, T}, w(\tau))$ of the player P on the level I of it control process, which formed due from the solution of the problems 1 and 2 it is required to find the set $\hat{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot)) \subseteq \mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot)) \subseteq \mathbf{V}(\overline{\tau, T}; u^{(e)}(\cdot))$ of the optimal minimax program controls $\hat{v}^{(e)}(\cdot) \in \mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot))$ of the player E on level II of the control of this control process and number $c_\beta^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot))$ of optimal value of the result of the minimax program control for the player E on the level II of the control of this control process for considered dynamical system and corresponding the control $u^{(e)}(\cdot)$ to the player P and these determines by the following relations:

$$\hat{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot)) = \{\hat{v}^{(e)}(\cdot) : \ \hat{v}^{(e)}(\cdot) \in \mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot)), \ c_\alpha^{(e)}(\overline{\tau, T}, w(\tau)) =$$

$$= \max_{\substack{\xi(\cdot) \in \Xi(\overline{\tau, T}; u^{(e)}(\cdot)) \\ \xi^{(1)}(\cdot) \in \Xi^{(1)}(\overline{\tau, T})}} \alpha((\tau), u^{(e)}(\cdot), \hat{v}^{(e)}(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) = \min_{v^{(e)}(\cdot) \in \mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot))} \ \max_{\substack{\xi(\cdot) \in \mathbf{W}(\overline{\tau, T}) \\ \xi^{(1)}(\cdot) \in \mathbf{W}^{(1)}(\overline{\tau, T})}} \alpha(w(\tau), u^{(e)}(\cdot), v^{(e)}(\cdot), \xi(\cdot), \xi^{(1)}(\cdot))\}; \tag{16}$$

$$c_\beta^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot)) = \max_{\xi^{(1)}(\cdot) \in \Xi^{(1)}(\overline{\tau, T})} \beta(w^{(1)}(\tau), \hat{v}^{(e)}(\cdot), u^{(e)}(\cdot), \xi^{(1)}(\cdot))\} =$$

$$= \min_{v(\cdot) \in \mathbf{V}(\overline{\tau, T}; u^{(e)}(\cdot))} \ \max_{\xi^{(1)}(\cdot) \in \Xi^{(1)}(\overline{\tau, T})} \beta(w^{(1)}(\tau), v(\cdot), u^{(e)}(\cdot), \xi^{(1)}(\cdot)). \tag{17}$$

Note, that we can consider the solutions of formulated problems 1–3 which in union are determined the solution of the main problem of two-level hierarchical minimax program control by the final states of the objects I and II for the discrete-time dynamical system (1)–(5) in the presence of perturbations.

ALGORITHM FOR SOLVING THE PROBLEMS 1–3

Thus, for any fixed and admissible time interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$ and realization τ-position $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = \{0, y_0, z_0\} = w_0 \in \mathbf{W}_0)$ of the player P on the level I of the control process and corresponding to it τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = \{0, z_0\} = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ of the player E on the level II of this two-level hierarchical control system for the discrete-time dynamical system (1)–(5) we can describe the algorithm for solving Problems 1–3 formulated above.

Then for fixed collection $(\tau, z(\tau), u(\cdot), v(\cdot)) \in \{\tau\} \times \mathbb{R}^s \times \mathbf{U}(\overline{\tau, T}) \times \mathbf{V}(\overline{\tau, T}; u(\cdot))$ we define by virtue of (2) and (4) the following set:

$$\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T) = \{z(T): \ z(T) \in \mathbb{R}^s, \ z(t+1) = f(t, z(t), u(t), v(t), \xi^{(1)}(t)), \ \xi^{(1)}(t) \in \mathbf{\Xi}_1^{(1)}(t), \ t \in \overline{\tau, T-1}\}, \quad (18)$$

where $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$ is a reachable set [1] of all admissible phase states of the object II at time moment T corresponding to the collection $(\tau, z(\tau), u(\cdot), v(\cdot))$.

We fix time interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$, admissible on the level II in the two level hierarchical control system for dynamical system (1)–(5) realization τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ of the player E and any admissible realization of the program control $u(\cdot) \in \mathbf{U}(\overline{\tau, T})$ of the player P on the level I of this control process.

Then, on the basis of the above definitions and results of the works [3], [4] the procedure of the construction the solution of the Problem 1 for the discrete-time dynamical system (1)–(5) can be represented as a sequence consisting from solving of the following three sub-problems:

1) constructing for every admissible control $v(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot))$ of the player E of the reachable set $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$ (note, that this set can be constructed by finding a solutions of a finite sequence a linear mathematical programming problems, and this set is convex, closed and bounded set in the space $\mathbb{R}^s$ [3]–[**?**]);

2) maximizing of the convex terminal functional $\tilde{\beta}$ which is defined by the relations (9) and (10) on the set $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$, namely, the formation of the following number:

$$\kappa_\beta^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot), v(\cdot)) = \max_{z(T) \in \mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)} \hat{\beta}(z(T)) = \hat{\beta}(\tilde{z}^{(e)}(T)) =$$
$$= \beta(w^{(1)}(\tau), u(\cdot), v(\cdot), \tilde{\xi}^{(1,e)}(\cdot)) = \max_{\xi^{(1)}(\cdot) \in \ \mathbf{\Xi}^{(1)}(\overline{\tau, T})} \beta(w^{(1)}(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot)) \quad (19)$$

(note, that the solution of this problem is reduced to solving a convex mathematical programming problem [3]–[**?**]);

3) constructing of the set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ and the number $\tilde{c}_\beta^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ from solving the following optimization problem:

$$\tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) = \{\tilde{v}^{(e)}(\cdot): \ \tilde{v}^{(e)}(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot)), \ \tilde{c}_\beta^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) =$$
$$= \kappa_\beta^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot), \tilde{v}^{(e)}(\cdot)) = \min_{v(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot))} \kappa_\beta^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot), v(\cdot))\} \quad (20)$$

(note, that the set $\mathbf{V}(\overline{\tau, T}; u(\cdot))$ is a finite set at the space $\mathbb{R}^q$, and then the solution of this problem is reduced to solving the finite discrete optimization problem).

Taking into consideration (9), (10), (14), (18)–(20), and the conditions stipulated for the system (1)–(5), one can prove (analogy as in works [3], [4]), that the following assertion is valid.

Theorem 1. For fixed time interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$, admissible on the level II in the two level hierarchical control system for the discrete-time dynamical system (1)–(5) realization τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ of the player E and for every admissible realization of the program control $u(\cdot) \in \mathbf{U}(\overline{\tau, T})$ of the player P on the level I of the control system, the set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ of the admissible program controls $\tilde{v}^{(e)}(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot))$ of the player E on the level II of the control system and the number $\tilde{c}_\beta^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$

constructed from a finite number procedures of solving the linear and convex mathematical programming problems, and the finite discrete optimization problem, and the following equalities are true:

$$\tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) = \mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot));\ \tilde{c}^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) = c^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)), \tag{21}$$

where the set $\mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ and the number $c^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ determined by the relation (14).

Then from this assertion follows that a solution of the problem 1 for the discrete-time dynamical system (1)–(5) can be formed from a finite number procedures of solving the linear and convex mathematical programming problems, and the finite discrete optimization problem on the basis of construction of the set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ and the number $\tilde{c}^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$.

Next, consider the algorithm for solving the problem 2.

For fixed collection $(\tau, y(\tau), u(\cdot), v(\cdot)) \in \{\tau\} \times \mathbb{R}^r \times \mathbf{U}(\overline{\tau, T}) \times \mathbf{V}(\overline{\tau, T}; u(\cdot))$ we define by virtue of (1) and (3) the following set:

$$\mathbf{G}(\tau, y(\tau), u(\cdot), v(\cdot), T) = \{y(T):\ y(T) \in \mathbb{R}^r,$$
$$y(t+1) = A(t)y(t) + B(t)u(t) + C(t)v(t) + D(t)\xi(t)),\ \xi(t)) \in \boldsymbol{\Xi}_1(t),\ t \in \overline{\tau, T-1}\}, \tag{22}$$

where $\mathbf{G}(\tau, y(\tau), u(\cdot), v(\cdot), T)$ is a reachable set [1] of all admissible phase states of the object I at time moment T corresponding to the collection $(\tau, y(\tau), u(\cdot), v(\cdot))$.

We fix time interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$, admissible on the level I and II in the two level hierarchical control system for the dynamical system (1)–(5) realizations τ-positions $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = w_0 \in \mathbf{W}_0)$ and $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w^{(1)}_0 \in \mathbf{W}^{(1)}_0)$ of the players P and E, respectively.

Then, on the basis of the above definitions and results of the works [3], [4] the procedure of the construction the solution of the Problem 2 for the discrete-time dynamical system (1)–(5) can be represented as a sequence consisting from solving of the following three sub-problems:

1) constructing of the reachable set $\mathbf{G}(\tau, y(\tau), u(\cdot), v(\cdot), T)$ (note, that this set can be constructed by finding a solutions of a finite sequence one-step operations only, and it is convex, closed and bounded polyhedron (with a finite number of vertices) in the space $\mathbb{R}^r$ [3], [4]);

2) maximizing of the convex terminal functional $\tilde{\alpha}$ which is defined by the relations (6)–(8) on the sets $\mathbf{G}(\tau, y(\tau), u(\cdot), v(\cdot), T)$ and $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$, namely, the formation of the following number:

$$\lambda^{(e)}_{\alpha}(\overline{\tau, T}, w(\tau), u(\cdot), v(\cdot)) = \mu \cdot < e, \tilde{y}^{(e)}(T) >_r + \mu^{(1)} \cdot \hat{\beta}(\tilde{z}^{(e)}(T)) =$$
$$= \max_{y(T) \in \mathbf{G}(\tau, y(\tau), u(\cdot), v(\cdot), T)} \mu \cdot < e, y(T) >_r + \max_{z(T) \in \mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)} \mu^{(1)} \cdot \hat{\beta}(z(T)) =$$
$$= \alpha(w(\tau), u(\cdot), v(\cdot), \xi^{(e)}(\cdot), \xi^{(1,e)}(\cdot)) = \max_{\substack{\xi(\cdot) \in \boldsymbol{\Xi}(\overline{\tau, T}) \\ \xi^{(1)}(\cdot) \in \boldsymbol{\Xi}^{(1)}(\overline{\tau, T})}} \alpha(w(\tau), u(\cdot), v(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) \tag{23}$$

(note, that the solution of this problem is reduced to solving a linear and a convex mathematical programming problems [3]), [4];

3) constructing of the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ and the number $\tilde{c}^{(e)}_{\alpha}(\overline{\tau, T}, w(\tau))$ from solving the following optimization problem:

$$\tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau)) = \{\tilde{u}^{(e)}(\cdot):\ \ \tilde{u}^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau, T}),\ \tilde{c}^{(e)}_{\alpha}(\overline{\tau, T}, w(\tau)) = \lambda^{(e)}_{\alpha}(\overline{\tau, T}, w(\tau), \tilde{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot)) =$$
$$\min_{\tilde{v}^{(e)}(\cdot) \in \tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \tilde{u}^{(e)}(\cdot))} \lambda^{(e)}_{\alpha}(\overline{\tau, T}, w(\tau), \tilde{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot)) = \min_{u(\cdot) \in \mathbf{U}(\overline{\tau, T})} \min_{\tilde{v}^{(e)}(\cdot) \in \tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))} \lambda^{(e)}_{\alpha}(\overline{\tau, T}, w(\tau), u(\cdot), \tilde{v}^{(e)}(\cdot))\} \tag{24}$$

(note, that the set $\mathbf{U}(\overline{\tau, T}; u(\cdot))$ is a finite set at the space $\mathbb{R}^p$, and the finite set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ constructed from (20), and then the solution of this problem is reduced to solving a finite discrete optimization problem).

Taking into consideration (6)–(8), (15), (20)–(24), and the conditions stipulated for the system (1)–(5), one can prove (analogy as in works [3], [4]), that the following assertion is valid.

Theorem 2. For fixed time interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$, admissible on the levels I and II in the two level hierarchical control system for the discrete-time dynamical system (1)–(5) realizations τ-positions $w(\tau) = \{\tau, y(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = w_0 \in \mathbf{W}_0)$ and $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w^{(1)}_0 \in \mathbf{W}^{(1)}_0)$ of the players P and E, respectively, the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ of the admissible program controls $\tilde{u}^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau, T})$ of the player P on the level I of the control system and the number $\tilde{c}^{(e)}_{\alpha}(\overline{\tau,T},w(\tau))$ constructed from a finite number procedures of solving the linear, and convex mathematical programming problems, and the finite discrete optimization problem, and the following equalities are true:

$$\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau)) = \mathbf{U}^{(e)}(\overline{\tau,T},w(\tau));\ \tilde{c}^{(e)}_{\alpha}(\overline{\tau,T},w(\tau)) = c^{(e)}_{\alpha}(\overline{\tau,T},w(\tau)), \tag{25}$$

where the set $\mathbf{U}^{(e)}(\overline{\tau,T},w(\tau))$ and the number $c^{(e)}_{\alpha}(\overline{\tau,T},w(\tau)))$ determined by the relation (15).

Then from this assertion follows that a solution of the Problem 2 for the discrete-time dynamical system (1)–(5) can be formed from a finite number procedures of solving the linear and con,vex mathematical programming problems, and the finite discrete optimization problem on the basis of construction of the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau))$ and the number $\tilde{c}^{(e)}_{\alpha}(\overline{\tau,T},w(\tau))$.

Then, on the basis of the above algorithms of solving the Problems 1 and 2 the procedure of constructing the solution of the Problem 3 for the discrete-time dynamical system (1)–(5) can be represented as a sequence consisting from solving of the following two sub-problems:

1) for any control $\tilde{u}^{(e)}(\cdot) \in \tilde{\mathbf{U}}^{(e)}(\overline{\tau,T})$ of the player P the constructing of the set $\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot))$ and the number $\bar{c}^{(e)}_{\alpha}(\overline{\tau,T},w(\tau))$ from solving the following optimization problem:

$$\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot)) = \{\bar{v}^{(e)}(\cdot):\ \bar{v}^{(e)}(\cdot) \in \tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot)),$$

$$= \bar{c}^{(e)}_{\alpha}(\overline{\tau,T},w(\tau)) = \lambda^{(e)}_{\alpha}(\overline{\tau,T},w(\tau),\tilde{u}^{(e)}(\cdot),\bar{v}^{(e)}(\cdot)) = \min_{\tilde{v}^{(e)}(\cdot)\in\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot))} \lambda^{(e)}_{\alpha}(\overline{\tau,T},w(\tau),\tilde{u}^{(e)}(\cdot),\tilde{v}^{(e)}(\cdot)) =$$

$$= \min_{u(\cdot)\in\mathbf{U}(\overline{\tau,T})}\ \min_{\tilde{v}^{(e)}(\cdot)\in\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),u(\cdot))} \lambda^{(e)}_{\alpha}(\overline{\tau,T},w(\tau),u(\cdot),\tilde{v}^{(e)}(\cdot)) =$$

$$= \lambda^{(e)}_{\alpha}(\overline{\tau,T},w(\tau),\tilde{u}^{(e)}(\cdot),\tilde{v}^{(e)}(\cdot)) = \mu\cdot\hat{\gamma}(\tilde{y}^{(e)}(T)) + \mu^{(1)}\cdot < e^{(1)},\tilde{z}^{(1,e)}(T) >_s =$$

$$= \max_{y(T)\in\mathbf{G}(\tau,y(\tau),\tilde{u}^{(e)}(\cdot),\tilde{v}^{(e)}(\cdot),T)} \mu\cdot\hat{\gamma}(y(T)) + \max_{z(T)\in\mathbf{G}^{(1)}(\tau,z(\tau),\tilde{u}^{(e)}(\cdot),\tilde{v}^{(e)}(\cdot),T)} \mu^{(1)}\cdot < e^{(1)},z(T) >_s\} \tag{26}$$

(note, that the sets $\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot))$ and $\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T})$ constructed from relations (24) and (20), respectively, and then the solution of this problem is reduced to solving a finite discrete optimization problem);

2) for any control $\tilde{u}^{(e)}(\cdot) \in \tilde{\mathbf{U}}^{(e)}(\overline{\tau,T})$ of the player P and any control $\bar{v}^{(e)}(\cdot) \in \bar{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot))$ of the player E the constructing of the number $\bar{c}^{(e)}_{\beta}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot))$ from solving the finite discrete optimization problem described by relation (19) and satisfies the following relation:

$$\bar{c}^{(e)}_{\beta}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot)) = \kappa^{(e)}_{\beta}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot),\bar{v}^{(e)}(\cdot)) =$$

$$=< e^{(1)},\bar{z}^{(1,e)}(T) >_s = \max_{z(T)\in\mathbf{G}^{(1)}(\tau,z(\tau),\tilde{u}^{(e)}(\cdot),\bar{v}^{(e)}(\cdot),T)} < e^{(1)},z(T) >_s . \tag{27}$$

Taking into consideration (18)–(27), and the conditions stipulated for the system (1)–(5), one can prove that the following assertion is valid.

Theorem 3. For fixed time interval $\overline{\tau,T} \subseteq \overline{0,T}$ $(\tau < T)$ and admissible on the level I of the control in two-level hierarchical dynamical system (1)–(5) realization the τ-position $w(\tau) = \{\tau,y(\tau),z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = \{0,y_0,z_0\} = w_0 \in \mathbf{W}_0)$ of the player P and admissible on the level II of the control process for this dynamical system the realization τ-position $w^{(1)}(\tau) \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w^{(1)}_0 \in \mathbf{W}^{(1)}_0)$ of the player E which formed due from the τ-position $w(\tau)$, and admissible realization of the program minimax control $u^{(e)}(\cdot) \in \tilde{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau)) = \mathbf{U}^{(e)}(\overline{\tau,T},w(\tau))$ of the player P on the level I of the control process, which formed due from the solution of the problems 1 and 2, the set $\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),u^{(e)}(\cdot)) \subseteq \mathbf{V}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),u^{(e)}(\cdot)) \subseteq \mathbf{V}(\overline{\tau,T};u^{(e)}(\cdot))$ of the admissible program controls $\bar{v}^{(e)}(\cdot) \in \bar{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),u^{(e)}(\cdot))$ of the player E on the level II of the control of this control process and the number $\bar{c}^{(e)}_{\beta}(\overline{\tau,T},w^{(1)}(\tau),u^{(e)}(\cdot))$ which form due from (26) and (27), respectively, constructed from a finite number procedures of solving the linear and the convex mathematical programming problems, and the finite discrete optimization problem, and the following equalities are true:

$$\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),u^{(e)}(\cdot)) = \hat{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),u^{(e)}(\cdot));$$

$$\bar{c}^{(e)}_{\alpha}(\overline{\tau,T},w(\tau)) = c^{(e)}_{\alpha}(\overline{\tau,T},w(\tau));\ \bar{c}^{(e)}_{\beta}(\overline{\tau,T},w^{(1)}(\tau),u^{(e)}(\cdot)) = c^{(e)}_{\beta}(\overline{\tau,T},w^{(1)}(\tau),u^{(e)}(\cdot)). \tag{28}$$

Then from this assertion follows that a solution of the Problem 3 for the discrete-time dynamical system (1)–(5) can be formed from a finite number procedures of solving the linear and convex mathematical programming problems, and the finite discrete optimization problems on the basis of construction of the set $\bar{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot))$ and numbers $\bar{c}_{\alpha}^{(e)}(\overline{\tau, T}, w(\tau))$ and $\bar{c}_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot))$.

Note, that on the basis of the above algorithm of solving the Problems 1–3 the procedure of the construction a solution of the main problem of two-level hierarchical minimax program control by the final states of the objects *I* and *II* for the discrete-time dynamical system (1)–(5) in the presence of perturbations can be formed from realization of a finite number procedures of solving the linear and the convex mathematical programming problems, and the finite discrete optimization problems.

CONCLUSION

Thus, we have presented the mathematical formalization of the main problem of two-level hierarchical minimax program control by the final states of the objects *I* and *II* for the discrete-time dynamical system (1)–(5) in the presence of external perturbations and with incomplete information. In this paper for solving of the investigated problem is proposed the algorithm that has a form of the recurrent procedure of solving a linear and a convex mathematical programming problems, and a finite optimization problems.

Results obtained in this paper are based on the studies [1]–[5] and can be used for computer simulation, design and construction of multilevel control systems for actual technical an economic dynamical processes operating under deficit of information and uncertainty. Mathematical models of such systems are presented, for example, in [1]–[3], [6]–[9].

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REFERENCES

[1] N. N. Krasovskii. *Theory of Motion of Control. Linear Systems*, Nauka, Moscow, 1968 [in Russian].

[2] N. N. Krasovskii, and A. I. Subbotin. *Game-Theoretical Control Problems*, Springer, Berlin, 1988.

[3] A. F. Shorikov. *Minimax Estimation and Control in Discrete-Time Dynamical Systems*, Urals State University Publisher, Ekaterinburg, 1997 [in Russian].

[4] A. F. Shorikov. Algorithm for Solving a Problem of ϵ-Optimal Program Control for Descrete-Time Dynamical System *Abstracts of International Seminar*. Ural State University, Ekaterinburg, Russia, June 22-26, 2005, pp. 176–178.

[5] M. S. Bazara, and C. M. Shetty. *Nonlinear Programming. Theory and Algorithms*, Wiley, New York, 1979.

[6] T. F. Filippova. *Estimates of reachable sets of impulsive control problems with special nonlinearity*, AIP Conference Proceedings, 1773 (100004), 2016, pp. 1-8.

[7] A. I. Propoy. *Theory elements of the optimum discrete processes*, Nauka, Moscow, 1973 [in Russian].

[8] B. Siciliano, and L. Villari *Robot Force Control*, Kluver Academic Publishers. Dordrecht, 1999.

[9] S. Tarbouriech, and G. Garcia *Control of Uncetain Systems with Bounded Inputs*, Springer-Verlag Lmd. London, 1997.